

# IV-1 A GENERAL THEORY FOR SPIN-WAVE SUPPRESSION IN FERRITES

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## 1. Introduction

When the microwave magnetic field applied to a ferrite polarised by a static magnetic field set to a value below that required for resonance exceeds a certain threshold value, unstable spin waves cause absorption of energy in the ferrite. This effect, termed the subsidiary absorption, manifests itself in a ferrite component as a non-linear increase in attenuation above a critical power level and can far exceed the attenuation at low power levels. There are two ways of suppressing such non-linear loss. One is to reduce the grain size of the ferrite so as to break up the unstable spin waves; the other achieves the same result by frequency modulation of the microwave field or amplitude modulation of the applied static field. These modulation methods are discussed in this paper, and for reasons outlined later, only the parallel-pump instability is considered.

Suppression of the subsidiary absorption by frequency modulation of the microwave field was suggested initially by Suhl<sup>1</sup> who derived the following criterion for suppression:

$$\left(\frac{h_c}{h_0}\right)^2 = \frac{1}{J_n^2(\beta)} \quad (1)$$

where  $h_c$  is the critical modulated field,  $h_0$  is the critical unmodulated field, and  $\beta$  is the modulation index. This theory was later improved by Morgenthaler<sup>2</sup>, who suggested that frequency modulation has the effect of channeling power into sidebands, each of which has an amplitude less than that of the unmodulated carrier, so increasing the instability threshold. This theory gave rise to the suppression criterion:

$$\left(\frac{h_c}{h_0}\right)^2 = \min \left( \frac{1}{J_n^2(\beta)} \right) \quad (2)$$

where integer  $n$  is adjusted to make the expression a minimum. The theories of Suhl and Morgenthaler were verified by Hartwick, Peresini and Weiss<sup>3</sup>, and Ollom and Goldstein<sup>4</sup> respectively for modulation frequencies  $f_m \gg \gamma \Delta H_k$  only, where  $\Delta H_k$  is the spin-wave linewidth, and  $\gamma$  is the gyromagnetic ratio. It is the object of this paper to present a theory which is valid for all values of  $f_m$ , and to do this allowance must be made for the linewidth of the non-linear process.

## 2. The General Suppression Criterion

Suhl's and Schlomann's<sup>5</sup> criterion for instability of a spin wave of frequency  $\omega_k$  is given by:

$$\eta_k^2 = h_c^2 R_k^2 - \left( \omega_k - \frac{\omega}{2} \right)^2 \quad (3)$$

where  $\eta_k = \gamma \Delta H_k / 2$ ,  $h_c$  is the critical microwave field magnitude, of frequency  $\omega$ , and  $P_k$  is a function of  $k$ . The critical field for a spin wave, frequency  $\omega_k$ , and microwave field, frequency  $2\omega_k$ , is therefore given by:

$$h_k^2 = \frac{\eta_k^2}{P_k^2} \quad (4)$$

Similarly, the critical magnitude,  $h_c$ , of a microwave field of frequency  $\omega$ , for a spin wave of frequency  $\omega_k$  is given by:

$$h_c^2 = \frac{\left(\omega_k - \frac{\omega}{2}\right)^2 + \eta_k^2}{P_k^2} \quad (5)$$

$$\text{i.e. } h_c^2 \left[ \frac{\eta_k^2}{\left(\omega_k - \frac{\omega}{2}\right)^2 + \eta_k^2} \right] = \frac{\eta_k^2}{P_k^2} = h_k^2 \quad (6)$$

This equation suggests that at the threshold, the field  $h_c$ , frequency  $\omega$ , is "seen" by the  $k$ th spin wave to have an effective value given by:

$$h_c \sqrt{\frac{\eta_k^2}{\left(\omega_k - \frac{\omega}{2}\right)^2 + \eta_k^2}} \quad (7)$$

If several fields are present, with magnitudes  $h_1, h_2, h_3, \dots, h_n$ , frequencies  $\omega_1, \omega_2, \omega_3, \dots, \omega_n$ , the  $k$ th spin-wave threshold is determined by the energy channeled into this spin wave from these fields; it is reasonable therefore, to postulate that at the threshold the effective values of these fields seen by the  $k$ th spin wave contribute to its amplitude so that

$$h_1^2 \left[ \frac{\eta_k^2}{\left(\omega_k - \frac{\omega_1}{2}\right)^2 + \eta_k^2} \right] + h_2^2 \left[ \frac{\eta_k^2}{\left(\omega_k - \frac{\omega_2}{2}\right)^2 + \eta_k^2} \right] + \dots + h_n^2 \left[ \frac{\eta_k^2}{\left(\omega_k - \frac{\omega_n}{2}\right)^2 + \eta_k^2} \right] = h_k^2 \quad (8)$$

This principle may be applied to the case of a frequency-modulated signal.

$$h = h_0 \cos(\omega_0 t + \beta \sin v_m t) \quad (9)$$

where  $\omega_0$  is the carrier frequency,  $\beta = 2\pi f_d / v_m$  is the modulation index,  $f_d$  is the deviation frequency and  $v_m = 2\pi f_m$ . By expanding equation (9) in terms of Bessel functions, and substituting the resulting frequency components in equation (8) we obtain:

$$\frac{h_0^2}{h_k^2} = \left\{ \sum_{n=1}^{\infty} J_n^2(\beta) \left[ \frac{\eta_k^2}{\left(\frac{x-nv_m}{2}\right)^2 + \eta_k^2} \right] + \sum_{n=0}^{\infty} J_n^2(\beta) \left[ \frac{\eta_k^2}{\left(\frac{x+nv_m}{2}\right)^2 + \eta_k^2} \right] \right\} \quad (10)$$

where  $x = 2\omega_k - \omega_0$ . Since  $h_0^2$  is proportional to the power in the applied signal and  $h_k^2$  to the threshold power of an unmodulated signal,  $h_0^2/h_k^2$  is therefore, a measure of spin-wave suppression of the  $k$ th spin wave. It is now only necessary to adjust the value of  $\omega_k$  to minimize equation (10) to determine the suppression ratio,  $(h_0^2/h_k^2)_{\min}$ , and the actual power handling improvement obtained,  $(h_0^2/h_k^2)_{\min}$ . This is best done with the aid of a digital computer.

For amplitude modulation of the static magnetic field equation (10) is modified only by changing the value of  $\beta$ , since as shown by Courtney<sup>6</sup> static field modulation is equivalent to applying a frequency modulated signal

$$h = h_0 \cos(\omega_0 t + \beta' \sin v_m' t) \quad (11)$$

where  $\beta' = 2h_m \frac{\partial \omega_k}{\partial H} / f_m'$ ,  $h_m$  is the amplitude of the static field modulation,  $H$  is the static field and  $f_m' = v_m' / 2\pi$  is the modulation frequency. The case of combined static field amplitude modulation and signal frequency modulation can now be considered by writing the microwave field as

$$h = h_0 \cos(\omega_0 t + \beta \sin v_m t + \beta' \sin v_m' t) \quad (12)$$

By expanding this equation in terms of Bessel functions, an expression for the suppression ratio is readily obtained as described above for frequency modulation alone. It is clear from these examples that this theory can be extended to apply to any modulation waveform.

### 3. Experimental Results

Consideration of equation (10) shows that it reduces to the criteria of Suhl and Morgensthaler for  $f_m \gg \gamma \Delta H_k$ , as is indicated in Fig. 1 which compares the three criteria discussed in this paper. It is seen that the theory presented here predicts an optimum modulating frequency for each suppression ratio, at which the deviation frequency is a minimum (approx  $\gamma \Delta H_k$ ), a fact observed experimentally by Ollom and Goldstein for a single-crystal YIG sphere. In Fig. 2 these experimental results are compared with the criterion of equation (10), and excellent agreement is observed over the range of values of  $f_d$  and  $f_m$  considered. Fig. 3 compares experimental results obtained by the authors for amplitude modulation of the static field of a single-crystal YIG sphere, with the criterion of equation (10) modified by equation (11). Again, good agreement is observed despite the difficulty of measuring the magnitude of the varying magnetic field.

In order to investigate the validity of the criterion of equation (10) in the case of waveguide components utilizing polycrystalline ferrite, the effect of static field modulation was investigated experimentally for a Reggia-Spencer phase-shifter (longitudinally-magnetized rod in rectangular waveguide) and a non-reciprocal phase-shifter (transversely-magnetized slab in rectangular waveguide), both using magnesium-manganese ferrite with saturation magnetisation 1150 gauss. 1-  $\mu$ sec pulses with a peak power of 100 KW at 5.4 Gc/s were applied to the component under test via a power divider, and the onset of the subsidiary absorption was determined by observing the formation of a spike on the leading edge of the pulse after transmission through the ferrite. Perpendicular pumping was assumed and the spin-wave linewidth estimated from the time constant associated with the spike.

Results for the Reggia-Spencer phase-shifter and the non-reciprocal phase-shifter are shown in figures 4 and 5 respectively. The good agreement between theory and experiment suggests that equations (10) and (11) can be used with confidence to predict spin-wave suppression in practical devices, and justifies the assumption of perpendicular pumping, i.e. the transverse microwave magnetic field is responsible for the subsidiary absorption.

From a practical point of view, the modulating fields and frequencies required to obtain significant improvement in the power-handling capability of a polycrystalline ferrite device are probably inconveniently large, e.g.  $f_m = 4$  Mc/s,  $h_m = 0.6$  oersted (peak) for 3 db improvement. Certain cases, however, may justify the extra equipment required to provide the static-field modulation as, for example, when there is a need to extend the power-handling capability of an existing ferrite component.

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### References

1. SUHL, H.: "Restoration of Stability in Ferromagnetic Resonance", Phys. Rev. Letters, Vol. 6, No. 4, Feb. 1961, p. 174.
2. MORGENTHALER, F.R., OLSEN, F.A. and BENNETT, G.E.: "Suppression of Spin Wave Instabilities Associated with Ferromagnetic Resonance", Proc. Int. Conf. on Magnetism and Crystallography, 1961, Vol. 1, p. 411.
3. HARTWICK, T.S., PERESSINI, E.R. and WEISS, M.T.: "Suppression of Subsidiary Absorption in Ferrites by Modulation Techniques", Phys. Rev. Letters, Vol. 6, No. 4, Feb. 1961, p. 176.
4. OLLOM, J.F. and GOLDSTEIN, H.L.: "Relation of Spin Wave Linewidth to Optimum Modulating Frequency Required for Suppression of Subsidiary Resonance in Ferrites", App. Phys. Letters, Vol. 2, No. 9, May, 1963, p. 170.
5. SCHLOMANN, E.: "Ferromagnetic Resonance at High Power Levels". Raytheon Tech. Report, R. 48, October, 1959.
6. COURTYNE, W.E.: Ph.D. Thesis, Queens University Belfast, 1963.

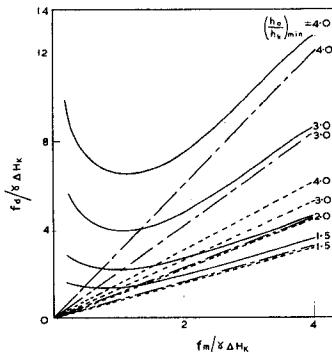


FIG. 1 - Comparison of the three Suppression Criteria

— Suhl  
— Morgensthaler  
— The Authors

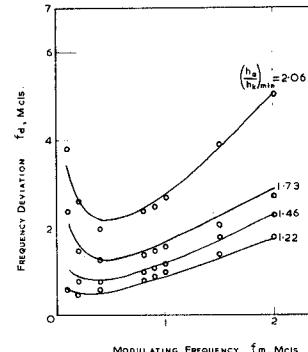


FIG. 2 - Microwave Field Frequency Modulation, Suppression Curves with Experimental Points of Ollom and Goldstein

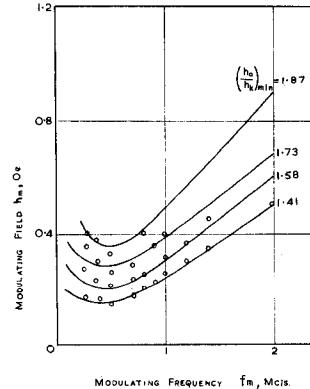


FIG. 3 - Static Field Amplitude Modulation, Suppression Curves with the Author Experimental Points

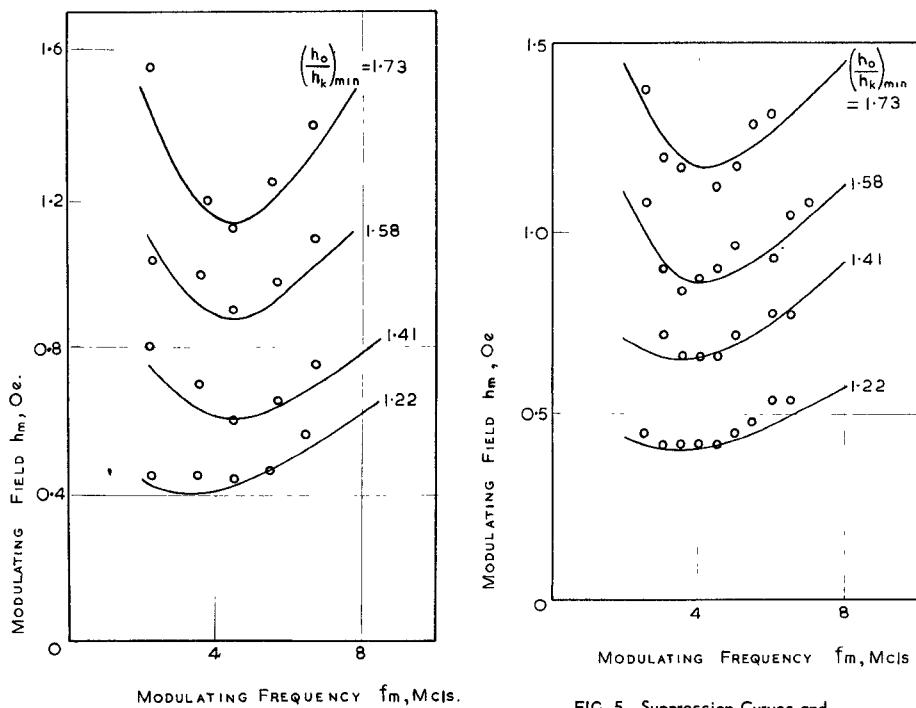


FIG. 5 - Suppression Curves and Experimental Points for Non-Reciprocal Phase-shifter

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